USING CONTROL THEORY TO EXPLORE THE BULLWHIP EFFECT IN THE SUPPLY CHAIN FOR A DETERIORATING PRODUCT

Jenhung Wang* and Pei-Hao Wang
Department of Logistics Management
National Kaohsiung First University of Science and Technology
Kaohsiung (824), Taiwan

ABSTRACT

The bullwhip effect phenomena in supply chain management were discussed a lot in the literatures. Most of them provided empirical evidences or analytical formulas to demonstrate the existence of the bullwhip effect in a simple two or more echelons supply chain. This paper will explore the phenomena in a quite different situation: a supply chain with deteriorating products. Unlike the conventional statistical or time series approaches, the control theory methodology was used to investigate the bullwhip effect of deteriorating products. It shows that the bullwhip effect was demonstrated effectively by using the control theory approach. The bullwhip effect could be reduced if the deterioration rate is high.

Keywords: Control Theory, Bullwhip Effect, Supply Chain Management

1. INTRODUCTION

It is well known that the balance between the productions and sales is always the most crucial issue in the supply chain management. The production rate often fluctuates more widely than does the actual sales rate. The longer the supply chain, the worse the problem is. Due to the inevitable demand uncertainty, it seems very difficult to match the demand and supply time by time. As the demand fluctuates, the order variability increases as one moving up the supply chain. This variability amplification is commonly known as the bullwhip effect, which was initially observed and investigated by Forrester [5]. The well-known Beer Game from MIT gave a more practical exercise of the bullwhip effect in the classrooms [19]. The bullwhip effect phenomenon is shown in Figure 1. Jacobs [8] implemented the Beer Game over the internet. In the MIT Beer Game, order/production quantities have to be determined for every member of a supply chain. However, only the retailer can observe the real customer demands, other members like wholesaler, distributor, and manufacturer have to make their own decisions based on incoming orders. Therefore, demand amplifications can be observed in the game. Sterman [19] summarized 3 key indicators of the bullwhip effect:

1. Oscillation: the orders may change up and down over time.
2. Amplification: the amplitude and variance of orders increases steadily from customer to retailer to factory.
3. Phase lag: the order rate tends to peak later as one moves from the retailer to the factory. The phase lag comes from order delays.

![Figure 1: Bullwhip effect (from bottom to top: retailer, wholesaler, distributor, factory) [19]](image)

Lee [11,12] proposed four major causes of the bullwhip effects: demand signals processing, the rationing game, ordering batching, price variations. Metters [14] used statistical tools to establish an empirical lower bound on the profitability impact of the bullwhip effect. He found that a decrease in demand seasonality could reduce the bullwhip effect...
and increase the profits. Kelle and Milne [10] investigated three activities in a supply chain: the purchase order of individual retailers, the aggregated orders of the retailers, and the production policy of suppliers. They illustrated how demand correlation can decrease the variability of aggregate orders, and how autocorrelation in buyer’s orders can smooth the supplier’s order policy. Chen et al. [1, 2] have quantified the bullwhip effect for order-up-to policies based on exponential smoothing forecasts as well as p-period moving average forecasts of demand. Xu et al. [23] used simple exponentially weighted moving average to forecast demands. They advocated that supply chain coordination could reduce the bullwhip effect. They also indicated that the manufacturer is the main beneficiary of coordination in terms of safety stock and resource waste reduction.

Deterioration of inventory is a common phenomenon in the real world. Ghare and Schrader [7] considered the case in which the time to deteriorate an item could be described by a negative exponential distribution, and a single EOQ model was developed. They also classified the characteristics of deterioration in three categories: (1) direct spoilage, like agricultural products, meats, food, etc.; (2) physical depletion or evaporation, like gasoline and alcohol; (3) radiation decay, chemical reaction. Deterioration of inventory can also be viewed in the point of market values. Raafat [15] classified products into three categories by the time value: (1) value unchanged over time, that is the conventional thinking of inventory management; (2) value increasing over time, like some antiques; (3) value decreasing over time, like food, electronic components and products.

Max [13] developed a production lot size model that incorporated an unfilled-order backlog for an inventory system with exponential decaying items. The approximate expression was obtained for the optimum production lot size, production cycle time, and total cycle time. Kang and Kim [9] presented a modified model to determine the price and production level for a deteriorating inventory system. The exponential distribution was used to represent the distribution of the time to deterioration. The optimal production quantity is derived under conditions of continuous review, deterministic demand and no shortage. Wee [22] developed a deterministic inventory model with quantity discount, pricing and partial backordering model for the deteriorating inventory. Rau et al. [16] developed a multi-echelon inventory model for a deteriorating item and derived an optimal joint total cost from an integrated perspective among the supplier, the producer, and the buyer.

Although abundant research results about the bullwhip effect and the inventory deterioration problems in the literatures, they were mostly discussed separately as two independent research topics. However, we know that the bullwhip effect exists everywhere of supply chains of any kind of products, and the inventory deterioration is also happened in most of products (if we adopt the idea of [15]). Besides, the supply chain management problems are getting so complex in the 21st century due to the global logistics operations, and not only the values of the agriculture products but also those of high technological electronic ones decline very quickly because of the high business competitions and short life times. Therefore, the bullwhip effect and inventory deterioration can happen simultaneously in many environments. It is necessary to construct a complete model to investigate both of the problems at one time.

This paper will construct the model to explore the bullwhip effect of a deteriorating product. Based on the system dynamic point of view, the bullwhip effect of products with or without deteriorating rate can be demonstrated and compared. Unlike the conventional statistical or time series approaches, the control theory methodology was used to demonstrate the bullwhip effect and investigate the influences of the deterioration rate. The conventional methodologies can be viewed as static approaches: the differences between the demand and order variances showed and compared by the analytic formulas. The control theory approach is a dynamic one because the signal fluctuations over time were depicted on the Figures.

The remainder of the paper is organized as follows: In section 2, an overview of the control theory approach is given. In section 3, a control system model to describe the dynamic behavior of productions and inventories for a deteriorating product is constructed. The system responses of the model and the influences of the deterioration rate on the bullwhip effect are analyzed in section 4. Finally, the conclusions of the model behaviors and related future research directions are given.

2. SUPPLY CHAIN DYNAMICS

In dealing with the bullwhip effect of the supply chain, there are two different approaches used. One is the statistical methodology, like [12,14], etc.; the other is the control system engineering. From a control system engineering point of view, a supply chain can be treated as a system with components of customers, retailers, distributors, wholesalers, and manufacturers. The behaviors and activities of a system are often complex and changing over time, that is, dynamic. In the supply chain example, activities include production scheduling, inventory management, information processing, distribution, transaction, etc.
2.1 System Point of View

The first step to establish a system is to clarify the components and boundary of the system. In the supply chain example, the system boundary can be defined based on the subject interested. Based on certain system components, the activities among the components should be defined precisely. If only physical distribution problem is interested, the system may not include the monetary transaction part. In investigating a well-defined system, we use modeling and response concepts. Modeling is the process of describing the physical of virtual system in mathematical terms, and response refer to the solution of the Equations that describe the system.

The framework of the system is the input/output concept as shown in Figure 2. There are several input factors to a system, while there are also several output results from the system. A system point of view is to consider the whole problem as a system, then investigate the relationship and cause-and-effect nature of the system. Due to the size of system, dynamic behavior of inputs, and complex relationships among inputs and outputs, the work to analyze the system response could be very difficult.

![Figure 2: General system point of view](image)

2.2 Control Theory Applications

Two approaches to the deeper understanding of dynamic behavior of logistics systems have appeared in the history. The older established these uses of linear control theory to analyze single or multi-loop systems. There are many variants to this analytic approach, first introduced by [18] using Laplace transform concepts to a single loop continuous time system, and his work was soon extended to discrete systems by [21]. The alternative approach is to use a special methodology originally called Industrial Dynamics by the originator, but now often referred to System Dynamics [17] and even Management Dynamics [3].

Towill and Vecchio [20] described that the industrial dynamics modeling has potential uses in the following three distinctive phases of supply chain re-engineering:

1. Planning (i.e. what should we do?).
2. Implementation (i.e. how do we do it?).
3. Control (i.e. real-time action via decision support systems).

Dejonckheere et al. [4] applied control theory to induce bullwhip effect by using different forecasting methods in order-up-to inventory replenishment policy. They summarized the analogies between supply chain world and control engineering world as shown in Figure 3.

![Figure 3: Summary of control engineering based methodology](image)

3. MODEL FORMULATIONS

A control system model was developed to describe the dynamic behavior of productions and inventories for a deteriorating product. In the system we assume a zero time lag for production, so the production rate can be adjusted very quickly, that is, a real-time production system. The deterioration rate is assumed to be a constant percentage of the actual inventory level. The initial actual inventory level is assumed to be zero, so we want the desired inventory level to be zero or close to zero during the time horizon. How soon and how close the actual inventory level to be zero is a good criterion to evaluate the stability of the system, so as the magnitude of the bullwhip effect.

3.1 Block Diagram

We defined the notation used in the model as follows.

- $I_d$: Desired inventory level
- $I_o$: Actual inventory level
- $\varepsilon$: Error ($I_d - I_o$)
- $D_L$: Market demand rate
- $\mu$: Production rate
- $\varphi$: Inventory deterioration rate
- $t$: Time
- $K_1$, $K_2$, $K_3$: Linear operators

As shown in Figure 4, the designed production-inventory control system is a closed-loop block diagram. The input signal is the desired inventory level $I_d$, and the output signal is the actual inventory level $I_o$. The information of the actual inventory level was fed back to the compare with the desired inventory level. The difference between the desired and actual inventory level is the error $\varepsilon$, which will determine the magnitude of production rate $D_L$. The demand signal $D_L$ could be several different types. The production rate is desired to satisfy both the demand $D_L$ and the desired inventory level $I_d$ as soon as possible.
According to the design of Figure 4, the actual inventory level can be represented as Equation (1). The actual inventory level at time t equals to the integral of the production rate minus both the demand rate and the inventory deterioration rate from time 0 to time t. Therefore, it is obvious that the linear operator K1 should be the integral operator.

\[ I_o(t) = K_1[\mu(t) - D_o(t) - K_2 I_o(t)] \]  

The relationship between the production rate and the inventory difference (error) can be shown as Equation (2), where \( K_3 \) is the decision rule to be decided in order to satisfy the demand rate and desired inventory level simultaneously.

\[ \mu(t) = K_3[\varepsilon(t)] \]  

From Equation (3) we can see the difference between the actual and desired inventory level is exact the error.

\[ \varepsilon(t) = I_o(t) - I_o(t) \]  

In Equation (1) the deterioration rate \( K_2 \) is used to measure the decay rate of the actual inventory level. Because the deterioration rate is assumed to be a constant percentage of the actual inventory level, we can set \( K_2 \) as a constant as shown in Equation (4)

\[ K_2 = \varphi \]  

The dynamic behavior of the whole system can be depicted by the system Equation (1) to (4). To analysis the system dynamics, the better approach is to apply the Laplace transformation to the Equations. For any function \( f(t) \) in time domain can be transferred to the complex domain \( F(s) \) by using Equation (5).

\[ F(s) = \int_0^\infty f(t)e^{-st}dt \]  

The Laplace transform of the derivative of a function \( f(t) \) is given by Equation (6). Because we assume that all system parameters are zero before time 0 (i.e., the system initiates from time 0), the term \( f(0) \) in Equation (6) will be zero for all variables.

\[ \mathcal{L}\{\frac{\partial}{\partial t} f(t)\} = sF(s) - f(0) \]  

By using Equation (5) and (6), we can transfer the system Equation from (1) through (3) to Equation (7) through (9). That is, we transfer the systems functions from time domain to complex domain.

\[ I_o(s) = \frac{1}{s}[\mu(s) - D_o(s) - \varphi I_o(s)] \]  

\[ \mu(s) = K_3(s)\varepsilon(s) \]  

\[ \varepsilon(s) = I_o(s) - I_o(s) \]  

In this production-inventory system, we concern about the relationship between the demand rate signal and the actual inventory level. We are interested in the dynamic behavior of the relationship. Therefore, it is convenient to observe the system behavior if we can transfer the original system in Figure 4 into the simple version in Figure 5.

As shown in Figure 5, the input signal \( D_o(s) \) was transferred to the output signal \( I_o(s) \) by the transfer function \( G(s) \). Therefore, the relationship between input and output signal can be shown in Equation (10).

\[ G(s) = \frac{I_o(s)}{D_o(s)} \]  

By taking algebraic operations on Equation (7) through (9), the transfer function of the system can be acquired as shown in Equation (11).

\[ G(s) = \frac{I_o(s)}{D_o(s)} = \frac{-1}{s + K_3(s) + \varphi} \]  

Equation (11) describes the dynamic behavior of the system clearly and completely. Because the initial values of the system variables are all zero, it is important to make sure that the actual inventory level approaches zero as the time going to infinity. We applied the Final Value Theorem as shown in Equation (12) to the transfer function (11). The result is shown in Equation (13).

\[ \lim_{t \to \infty} f(t) = \lim_{s \to 0^+} sF(s) \]  

\[ \lim_{t \to \infty} f(t) = \lim_{s \to 0^+} sF(s) \]
\[ L_{\infty} I_o(t) = \lim_{s \to 0} \frac{-sD_L(s)}{s + K_3(s) + \phi} \quad (13) \]

In Equation (13), for different input signal \( D_L(s) \), a good production rate \( K_3(s) \) should be found to ensure that the actual inventory level will go to zero as the time goes to infinity. We discuss different demand rates and their corresponding production rates as follows.

### 3.2 Step Function Input

The simplest demand function is the step function, which means that the demand goes up from zero to one on time zero and keep the value of one as time goes to infinity. Equation (14) shows the mathematical form of step function. The Laplace transform of step function is shown as Equation (15).

\[ D_L(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases} \quad (14) \]

\[ \mathcal{L} [D_L(t)] = D_L(s) = 1/s \quad (15) \]

Substitute Equation (15) into Equation (13), the result is shown in Equation (16).

\[ \lim_{s \to 0} I_o(t) = \lim_{s \to 0} \frac{-1}{s + K_3(s) + \phi} \quad (16) \]

To make sure that the actual inventory level to be zero eventually, a good design rule of \( K_3(s) \) should be found such that the denominator of Equation (16) will approach infinity as \( s \) approaches zero. An appropriate design of \( K_3(s) \) is shown as Equation (17) [18]. The corresponding production rate can be found directly as shown in Equation (18).

\[ K_3(s) = \frac{1}{s^2}(a + bs), \quad k \geq 1, \quad a > 0, \quad b > 0 \quad (17) \]

\[ \mu(s) = K_3(s) \epsilon(s) = \left( \frac{a}{s^2} + \frac{b}{s^3} \right) \epsilon(s) \quad (18) \]

The production rate function in time domain can be found if we take the inverse Laplace transform of Equation (18). Two examples are given as follows.

(i) for \( k=1, \) then \( \mu(s) = \left( \frac{a}{s^2} + \frac{b}{s} \right) \epsilon(s), \) by inverse Laplace transform:

\[ \mu(t) = \int a \epsilon(t) dt + b \int \epsilon(t) dt \quad (19) \]

(ii) for \( k=2, \) then \( \mu(s) = \left( \frac{a}{s^2} + \frac{b}{s^3} \right) \epsilon(s), \) by inverse Laplace transform:

\[ \mu(t) = \int \int a \epsilon(t) dt + b \int \epsilon(t) dt \quad (20) \]

### 3.3 Power Function Input

Another common demand function is the power function as shown in Equation (21). The Laplace transform of power function is shown as Equation (22).

\[ D_L(t) = \begin{cases} 0, & t < 0 \\ t^n, & t \geq 0 \end{cases} \quad (21) \]

\[ \mathcal{L} [D_L(t)] = D_L(s) = \frac{n!}{s^{n+1}} \quad (22) \]

Substitute Equation (22) into Equation (13), the result is shown in Equation (23).

\[ \lim_{s \to 0} I_o(t) = \lim_{s \to 0} \frac{-n!}{s^{n+1} + s^n K_3(s) + s^a \phi} \quad (23) \]

We need to find an appropriate design of \( K_3(s) \) such that the value of Equation (23) will approach zero. The design of \( K_3(s) \) is shown as Equation (24) [18]. The corresponding production rate can be found directly as shown in Equation (25).

\[ K_3(s) = \frac{1}{s^2}(a + bs), \quad k \geq n+1, \quad a > 0, \quad b > 0 \quad (24) \]

\[ \mu(s) = K_3(s) \epsilon(s) = \left( \frac{a}{s^2} + \frac{b}{s^3} \right) \epsilon(s) \quad (25) \]

The production rate function in time domain can be found if we take the inverse Laplace transform of Equation (25). Two examples are given as follows.

(i) for \( n=1, \) \( k=2, \) then \( \mu(s) = \left( \frac{a}{s^2} + \frac{b}{s} \right) \epsilon(s), \) by inverse Laplace transform:

\[ \mu(t) = \int \int a \epsilon(t) dt + b \int \epsilon(t) dt \quad (26) \]

(ii) as \( n=2, \) \( k=3, \) then \( \mu(s) = \left( \frac{a}{s^3} + \frac{b}{s^4} \right) \epsilon(s), \) by inverse Laplace transform:

\[ \mu(t) = \int \int \int a \epsilon(t) dt + b \int \epsilon(t) dt \quad (27) \]

### 4. SYSTEM RESPONSE ANALYSIS

Based on the transfer functions and designs of production rate functions developed in previous section, we are able to observe the dynamic behavior and the bullwhip effect of the system. Different
input signals with different production rate parameters and different deterioration rate are tested for the system.

4.1 Step Response
With the step function input and the design of $K_3(s)$ in Equation (17), the response of the output signal of Equation (11) in time domain can be observed. Figure 6, 7, and 8 show the response of the system with deterioration rate equals to 0.1 and different values of parameters $a$ and $b$ in the production rate function. The “Amplitude” in Figures represents the magnitude of the output signal in the time domain, that is, the actual inventory level. Because the demand rises from zero to one at time zero, it is reasonable that the actual inventory level will be negative in the beginning. The signal was fed back to compare with the desired inventory level, zero. Such a negative error signal will initiate the production activity immediately with the designed production rate function. The system was assumed a real-time production system, so the error signal can approach to zero very quickly, so as the actual inventory level.

The fluctuation of the response is a good index of bullwhip effect. Facing the negative inventory level, the production will start to supply the demand. However, even for the stable demand signal, step function, the actual inventory level fluctuate over time for a while. This fluctuation shows the inconsistency between the demand and production, which represents the so-called bullwhip effect. The more fluctuation in the response, the bigger bullwhip effect in the system. We can find that with larger values of parameters $a$ and $b$ of the production rate function, the bullwhip effect can be tamped. However, the bullwhip effect will never be eliminated from the system.

4.2 Unit-Ramp Response
For the power function response, we consider the simple case, unit-ramp function as shown in Equation (28) as the input function.

$$D_u(t) = \begin{cases} 0, & t < 0 \\ t, & t \geq 0 \end{cases} \quad (28)$$

Following the design of $K_3(s)$ in Equation (24) with $n=1$ & $k=2$, the response of the output signal of Equation (11) in time domain can be observed. Figure 9, 10, and 11 show the response of the system with deterioration rate equals to 0.1 and different values of parameters $a$ and $b$ in the production rate function. The fluctuation of output signal shows the existence of bullwhip effect. We can find the larger values of parameters $a$ and $b$ of the production rate function, the smaller bullwhip effect in the system. This result is consistent with that of the step response.
Figure 10: Unit-ramp response with $k=2$, $b=6$, $\varphi=0.1$, and $a=0.5, 0.1, 0.05$ respectively.

Figure 11: Unit-ramp response with $k=2$, $\varphi=0.1$, and $(a, b)=(0.5, 6), (0.1, 10), (0.05, 20)$, respectively.

The unit-ramp response is quite different from the step response. First, it takes much longer time to get stable. Second, it fluctuates very seriously in the transit state, that is, the bullwhip effect of unit-ramp response is much larger than that of step response. We explain this phenomenon as follows. For step function, the demand only changes from zero to one at time zero, so the production rate can be adjusted to satisfy the stable one unit demand very quickly. However, for unit-ramp function, the demand increases from zero at time zero, and causes the insufficiency of actual inventory level immediately. This signal is fed back to stimulate the production activity. Of course, it takes time to catch up the demand, so the actual inventory level must fluctuate for a while. Unlike the step response, the change of demand is not only happened at time zero, but in the whole time horizon. Therefore, the production rate has to change time by time from time zero to catch up the demand and satisfy the zero inventory goal simultaneously. This complex situation causes the system takes much longer time to get stable. Besides, the frequency of fluctuation is getting so high compared with the fluctuation frequency of step response. That means the system has to adjust itself many times.

Comparing the step response and unit-ramp response, we found the bullwhip effect will getting serious as the demand signal becomes complex.

4.3 The Role of Deterioration Rate

This research is highly interested in the impacts of the deterioration rate to the bullwhip effect of the production-inventory system. Figure 12 and 13 show the step response and unit-ramp response of the system respectively. We test different deterioration rates and both responses have the same results: the higher deterioration rate, the smaller bullwhip effect.

Figure 12: Step response with $k=1$, $a=10$, $b=1$, $\varphi=0.1, 0.3, 0.5$ respectively.

Figure 13: Unit-ramp response with $n=1$, $k=2$, $a=0.5$, $b=10$, $\varphi=0.1, 0.3, 0.5$ respectively.

It seems hard to Figure out these results intuitively. Deterioration usually gives us a negative impression because it always decreases the quantities or values of the products or inventories. High deterioration rate usually implies more damage to the business. However, here we found a different result: high deterioration causes small bullwhip effect, and business often prefers small bullwhip effect.

Why high deterioration rate can decrease the bullwhip effect? Remember the closed-loop production-inventory system is a self-adjusted system. The information of actual inventory level is fed back to compare with the desired inventory level to adjust the production rate instantly. The production quantities minus the demand and deterioration quantities equal to the actual inventory goal, then the actual inventory level information is fed back again. The process keep on going, so the system needs to adjust itself to keep balance among the production, demand, and inventory again and again. In the
self-adjusted process, the less uncertainties, the more stable system will be. The only uncertainty in the system is the demand function. The actual inventory level is composed of three parts: production quantity, demand quantity, and deterioration quantity. If the weigh of uncertainty is high, it is intuitive that the system need more time to stabilize the output signal, actual inventory level. Therefore, the high deterioration rate will decrease the weight of uncertainty in the composition of actual inventory level, and thus decrease the bullwhip effect.

5. CONCLUSIONS AND FUTURE STUDIES

In this paper we implemented the control theory methodologies to explore the bullwhip effect for deteriorating products. That is, we construct the model to discuss 2 famous research subjects: bullwhip effect and inventory deterioration, simultaneously. The dynamic behaviors of the production-inventory system were explored and viewed by the Figures. From the system point of view, we can verify the relationship between the input and output signals very clearly and efficiently. Besides, we showed the complex inter-correlated activities of the system, and how the closed-loop system can adjust itself automatically with the good design of linear operators.

This paper considers a simple system with constant deterioration inventory rate. Different demand functions were tested and the output signals and the bullwhip effects were compared. The results showed the more simple demand function, the smaller bullwhip effect. This result is rational. Complex demand signal will cause the system to adjust the production quantity very often and fast, thus the error between and desired and actual inventory levels cannot be reduced to zero quickly. The existence of deterioration rate will decrease the values of inventory and will make the system more complex. However, the bullwhip effect is tamed in the system with deterioration rate. This is a really surprising result. It stimulates us to re-explore the complexity of the supply chain dynamics.

For future studies, a system with production time delay can be investigated in detail. Besides, a discrete version of Laplace Transformation, the so-called z-transformation can be used to extend the components and activities of the system in the discrete time domain.

REFERENCES


**ABOUT THE AUTHORS**

**Jenhung Wang** is an assistant professor of the department of Logistics Management, National Kaohsiung First University of Science and Technology, Taiwan. He received his Ph.D. degree from the department of Industrial Engineering, Purdue University. His research interests include the bullwhip effect in supply chains, inventory management, optimization analysis, etc.

**Pei-Hao Wang** is studying in the master program in the department of Logistics Management, National Kaohsiung First University of Science and Technology, Taiwan.

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